Shannon Sampling Theorem in Information Theory and Its Application

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Abstract: The sampling theory and application is one of the hottest field in digital signal processing and digital communication. A signal f is said to be finite energy if $\|f\| < \infty$, where $\|\cdot\|$ is the square norm defined by $\|f\| = (\int_{\mathbb{R}} |f(t)|^2 dt)^{\frac{1}{2}}$. f is said to be bandlimited if $\hat{f}(\omega) = 0$ whenever $|\omega| > \Omega$ for some $\Omega > 0$, where $\hat{f}$ is the Fourier transform of f defined by $\hat{f}(\omega) = \int_{\mathbb{R}} f(t)e^{-it\omega}dt$. In this case, f is also called an Ω-band signal. The classical Shannon sampling theorem shows that any bandlimited signal f is uniquely determined by sampled values$\{f(\frac{k\pi}{\Omega})\}_{k\in\mathbb{Z}}$. Since in some practical systems sampling cannot be always made uniformly, one has to consider the non-uniformly sampled signal. On the other hand, bandlimited signal is not always realistic since a bandlimited signal is of infinite duration. The concept of shift-invariant spaces first arose in approximation theory and wavelet theory, which generalize the space of bandlimited functions and is more realistic for modeling signals with smoother spectrum, taking into account the real acquistion and reconstruction devices, and for the numerical implementation, shift-invariant spaces were studied in the last twenty years. The objective of the so-called sampling problem is to recover a signal(function) f from its samples $\{f(x_k)\}_{k\in\mathbb{Z}}$. Obviously this problem is ill-posed. So a successful reconstruction requires that signal(function) f belong to some signal spaces.